Year 13 Further Maths Curriculum Map

Sequencing of topics	What knowledge will students develop? (Including key terminology)	What skills will students develop? (Including literacy & numeracy)	Assessment opportunities	Homework opportunities	Personal development (Ursuline Values, Catholic Social Teaching, Cultural Capital, Cross-curricular, Careers)	Curriculum Links
Autumn Term						
Complex numbers	• Know and use $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$ • De Moivre's theorem • The nth roots of $z = re^{i\theta}$ and complex roots of unity	• be able to multiply and divide complex numbers in modulus-argument and exponential form; • know and use cosine and sine in terms of the exponential form. • understand, remember and be able to use de Moivre's theorem: $z^n = r^n e^{in\theta} = r^n(\sin n\theta + i\cos n\theta)$; • be able to derive multiple angle formulae/expressions e.g. $\cos 3\theta$ in terms of powers of $\cos\theta$, and $\sin 3\theta$ in terms of multiple angles of $\sin\theta$; • be able to apply de Moivre's	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **United in Harmony**: Teaching complex numbers emphasizes the interconnectedness of real and imaginary components. By exploring the unity and harmony of these components, students can develop a deeper appreciation for the beauty and elegance of complex numbers. • Call to Family, Community, and Participation • Solidarity • Availability of quality textbooks or online resources specifically dedicated to complex numbers. • Physics • Computer Science • Electrical engineer	Complex numbers in CP1

		theorem to sum a geometric series. • know how to solve completely equations of the form zn – a – ib = 0, giving special attention to cases where a = 1, b = 0 •			• Control systems engineer • Mathematician • Physicist • Software developer	
Hyperbolic functions	• sinh *x*, cosh *x*, tanh *x* and their inverses • Logarithmic forms of the inverse hyperbolic functions and integrate functions of the form $(x^2 + a^2)^{-\frac{1}{2}}$ and $(x^2 - a^2)^{-\frac{1}{2}}$	• know the definitions of sinh x, cosh x and tanh x including their domains and ranges; • be able to sketch graphs of the hyperbolic functions; • be able to differentiate and integrate the hyperbolic functions and know the standard results; • understand and be able to use the inverse hyperbolic functions including domains and ranges. • be able to derive, use and know the	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Discerning and Joyful** hyperbolic functions require students to discern patterns and relationships between the exponential and trigonometric functions. Hyperbolic functions have unique properties and behaviours that differ from their trigonometric counterparts. • Option for the Poor and Vulnerable • Care for God's Creation • Access to online tutorials or videos that offer explanations and	Trigonometric identities Calculus

		logarithmic forms of the inverse hyperbolic functions. •			demonstrations of hyperbolic functions. • Physics • Physicist • Mathematician • Actuary • Financial Analyst • Electrical Engineer	
Polar coordinates	• Convert between Cartesian and polar and sketch $r(\theta)$ • Area enclosed by a polar curve	• understand and be able to use polar coordinates and be able to convert between polar and Cartesian coordinates; • know how to sketch standard polar curves. • be able to find tangents parallel and perpendicular to the initial line; • be able to find (compound) areas under polar graphs using the formula $\frac{1}{2}\int r^2 \mathrm{d}\theta$	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Listening and Attentive** polar coordinates require students to actively listen and be attentive to the relationships between angles and distances. • Life and Dignity of the Human Person • Call to Family, Community, and Participation • Exposure to real-world examples where polar coordinates are used, such as in astronomy or navigation. • Physics • Geography • Surveyor • Astronomer • Marine Navigator	Calculus Trigonometry

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Further algebra and functions (series)	• Method of differences • Maclaurin series	• be able to use the method of differences to sum simple finite series. • be able to find and use higher derivatives of functions; • know how to express functions as an infinite series in ascending powers using Maclaurin's expansion; • be able to find the series expansion of composite functions.	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Discerning and Joyful** Students can appreciate the power of approximation and the elegance of expressing complex functions as infinite sums of simpler terms. • Option for the Poor and Vulnerable • Care for God's Creation • Availability of online tutorials or videos that demonstrate the process of finding Maclaurin series for different functions. • Physics • Computer Science • Economics • Electrical Engineer • Financial Analyst • Game Developer	Series Calculus
Further calculus	• Improper integrals • Mean value of a function • Integrate using partial fractions	• know how to deal with infinity as a limit of a definite integral; • be able to integrate	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos	• **Leading for Justice, Truth, and Integrity** Students can explore how calculus enables us to model and analyse real-world	Calculus Partial fractions Complex numbers

	• Differentiate inverse trigonometric functions and integrate using trigonometric substitutions • Further volumes of revolutions	functions across limits which include values when the function is undefined i.e. deal with discontinuous integrands. • understand and be able to evaluate the mean value of a function. • be able to integrate functions which can be split into partial fractions up to denominators with quadratic factors. • be able to differentiate inverse trigonometric functions such as $\frac{1}{2}$ arctan x2 ; • know how to integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and be able to choose trigonometric substitutions to		2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	phenomena accurately. By employing calculus, mathematicians and scientists can make informed decisions, solve problems, and gain a deeper understanding of the natural world. • Option for the Poor and Vulnerable • Solidarity • Exposure to diverse cultural contexts and perspectives in relation to advanced calculus techniques and their applications in various fields such as physics, engineering, and economics. • Physics • Economics • Computer Science • Aerospace Engineer • Economist • Data Scientist	

		integrate associated functions. • be able to derive formulae for and calculate volumes of revolution about both the x and y-axes. • be able to find volumes of revolution for functions given in parametric form				
Differential equations	• Integrating factors to solve first order differential equations • Second order differential equations of the form $y'' + ay' + by = f(x)$ • Modelling	• be able to identify the form of first order differential equations that can be solved by an integrating factor and carry out the solution; • be able to find general and particular solutions of differential equations of this form. • be able to solve second order differential equations of the form $y'' + ay' + by = f(x)$ where $f(x)$ is a polynomial, exponential or	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Discerning and Joyful** differential equations involve discerning patterns, relationships, and the behaviour of functions and their derivatives. By studying differential equations, students develop a discerning mindset and experience joy in unravelling the connections between different mathematical concepts. • Life and Dignity of the Human Person	Calculus

		trigonometric function; • be able to find general and particular solutions of second order differential equations of this form. • be able to use differential equations in modelling in kinematics and in other contexts; • be able to solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion; • be able to model damped oscillations using second order differential equations and interpret their solutions.			• Care for God's Creation • Exposure to diverse cultural contexts and perspectives in relation to differential equations and their applications in various fields such as physics, biology, engineering, and economics. • Physics • Biology • Economics • Biomedical Engineer • Environmental Scientist • Financial Risk Analyst	
Spring Term						
Algorithms and graph theory	• Introduction to algorithms	• understand what an algorithm is; • be able to trace an algorithm in the	End of term assessment	Textbook/Practice Book/Other online resources	• **United in Harmony** algorithms and graph theory highlight the	Quadratic

	• Sorting algorithms • Introduction to graph theory	form of a flow chart; • be able to trace an algorithm given as instructions written in text; • know how to determine the output of an algorithm and how it links to the input; • be able to determine the order of a given algorithm and standard network problems. • know how to apply a bubble sort algorithm to a list of numbers or words; • know how to apply the quick sort algorithm to a list of numbers or words, clearly identifying the pivots used for each pass; • be able to identify the number of comparisons and swaps used in a given pass;		These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	interconnectedness and harmony within mathematical structures and problem-solving techniques. Algorithms and graph theory provide tools to analyse and solve complex problems by representing them in a graphical format and applying algorithms to navigate and optimize these graphs. • Call to Family, Community, and Participation • Solidarity • Access to coding clubs or programming courses that focus on algorithm design and graph theory. • Computer Science • Information & Communications Technology • Sociology • Business Studies	

		• be able to identify size, efficiency and order of an algorithm and use them to make predictions; • know how to solve bin packing problems using full bin, first fit, and first fit decreasing algorithms, and understand their strengths and weaknesses. • know the meaning of the vocabulary used in graph theory e.g. degree of a vertex, isomorphic graphs, walks, paths and cycles; • be familiar with different types of graph e.g. complete, planar, isomorphic, simple, connected; • understand graphs represented in matrix form; • be familiar with k notation;			• Software Engineer • Network Administrator • Operations Research Analyst	

		• know the definition of a tree; • be able to determine if a graph is Eulerian, semi-Eulerian or neither, and find Eulerian cycles. •				
Algorithms on graphs I (part 1)	• Minimum connectors (spanning trees) • Dijkstra's algorithm	• understand the meaning of a minimum spanning tree; • be able to apply Kruskal's algorithm to a network to find the minimum spanning tree; • be able to apply Prims algorithm to a network to find the minimum spanning tree; • be able to apply Prim's algorithm to a distance matrix to find the minimum spanning tree. • be able to apply Dijkstra's algorithm to find the shortest path between two	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Loving and Compassionate** students develop a loving and compassionate mindset by recognizing the importance of finding efficient and optimal solutions. This promotes a caring attitude towards solving problems in an efficient and compassionate manner. • Call to Family, Community, and Participation • Care for God's Creation • Participation in math or computer science competitions that	Algorithms and graph theory

		vertices in a network; • be able to trace back through a network to be able to find the route corresponding to the shortest path; • be able to consider modifications to an original shortest path problem, for example by dealing with multiple start points or a different end point. •			include problems related to shortest path algorithms. • Physics • Biology • Economics • Data Analyst • Operations Manager • Supply Chain Analyst • Urban Planner • Software Developer	
Algorithms on graphs II (part 1)	Route inspection problem	• be able to determine whether a graph is traversable; • be able to apply an algorithm to solve the route inspection problem; • find a route by inspection; • understand the importance of the order of vertices	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **United in Harmony** The route inspection problem involves finding the shortest possible route that visits a set of specified locations and returns to the starting point. By studying this problem, students can appreciate the unity and harmony within mathematics,	Algorithms and graph theory

		of the graph in finding a route. •			understanding how different components of the problem interact and influence the optimal solution. • Life and Dignity of the Human Person • Call to Family, Community, and Participation • Exposure to real-world scenarios where the route inspection problem arises, such as logistics or delivery route optimization. • Geography • Economics • Computer Science • Business Studies • Transportation planner • Logistics manager • Urban planner • Geospatial analyst • Supply chain analyst • Data scientist	

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Linear programming	• Formulation of problems • Graphical solutions	• know how to formulate a linear programming problem from a real-life problem (write inequalities from worded questions); • be able to form an appropriate objective function to maximise or minimise. • know how to represent a linear programming problem graphically and identify the feasible region; • be able to solve linear programming problems to find a maximum or minimum; • be able to interpret solutions in the context of the original real life problem. •	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Discerning and Joyful** By exploring the properties of linear programming models, students develop a discerning mindset and experience joy in finding efficient ways to allocate resources, plan production, or solve optimization problems. • Option for the Poor and Vulnerable • Call to Family, Community, and Participation • Exposure to real-world applications of linear programming, such as resource allocation, production planning, or financial optimization. • Economics • Business Studies	Matrices

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					• Computer Science • Operations Research Analyst • Financial Analyst • Supply Chain Manager • Management Consultant • Data Scientist	
Critical path analysis (part 1)	• Activity networks; precedence tables • Critical path algorithm; earliest and latest event times • Total float; Gantt charts	• be able to model a project by an activity network from a precedence table; • be able to complete a precedence table from a given network; • understand the use of dummies. • know how to carry out a forward pass and backward pass using early and late event times; • be able to interpret and use dummies; • be able to identify critical activities and critical paths. • know how to determine the	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Listening and Attentive** By being attentive to the dependencies and constraints within a project, students can develop the skill of active listening and paying close attention to the critical elements that impact the overall timeline. • Call to Family, Community, and Participation • Option for the Poor and Vulnerable • Access to textbooks or online resources that provide step-by-step explanations and examples of critical path analysis.	Algorithms and graph theory

		total float of activities; • be able to construct and interpret Gantt (cascade) charts. •			• Business Studies • Economics • Design & • Technology • Project manager • Construction • manager • Event planner • Operations • manager • Supply chain • Manager • Civil Engineer	
Algorithms and graph theory (part 2)	Planarity algorithm	• be able to apply the planarity algorithm for planar graphs; • be able to determine if a graph contains a Hamiltonian cycle.	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Algorithms on graphs IOpportunities for flipped learning	• **Listening and Attentive** Learning planarity algorithms requires careful attention and listening to identify and understand the constraints that determine whether a graph can be drawn without any edge crossings. • Option for the Poor and Vulnerable • Life and Dignity of the Human Person • Access to online platforms or coding resources that provide tutorials and practice problems	Algorithms and graph theory

					related to graph algorithms. • Computer Science • Information & Communications Technology • Design & Technology • Art • Graph theorist • Network analyst • Data visualization specialist • Web designer • Software developer	
Algorithms on graphs I (part 2)	Floyd's algorithm	• be able to find all the shortest paths between all the pairs of vertices using Floyd's algorithm. •	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Discerning and Joyful** By applying this algorithm, students can experience joy in discovering optimal solutions and finding efficient ways to navigate through complex networks. • Option for the Poor and Vulnerable • Life and Dignity of the Human Person • Access to online platforms or coding resources that provide tutorials and practice problems	Algorithms and graph theory

					related to graph algorithms. • Computer Science • Information & Communications Technology • Physics • Network engineer • Data analyst • Operations research analyst • Transportation planner • Telecommunications specialist	
Algorithms on graphs II (part 2)	Travelling salesman problem	• understand the travelling salesman problem and that there is no simple algorithm to solve it for complex networks; • be able to use the nearest neighbour algorithm to find upper bounds for the problem; • be able to find lower bounds for a problem; • understand that not all upper and lower bounds	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **United in Harmony** By studying this problem, students can appreciate the harmony between optimization and decision-making in real-world scenarios. • Call to Family, Community, and Participation • Solidarity • Exposure to research projects or internships in computer science that involve algorithms on graphs, such as	Algorithms and graph theory

		give a solution to the problem; • know how to identify the best upper and lower bounds; • be able to solve the travelling salesman problem and interpret this solution in the context of the problem. •			network analysis or social network modeling. • Geography • Economics • Business Studies • Computer Science • Operations manager • Logistics coordinator • Supply chain analyst • Route planner • Travel industry specialist	
Linear programming (part 2)	• Formulation of problems • Simplex algorithm • Big-M and two-stage Simplex	• understand and use slack, surplus and artificial variables. • be able to use slack variables to write inequality constraints as equations; • know how to rewrite LP problems so that each equation contains all the variables x, y, s, and t; • be able to put the information in an initial tableau; • be able to find the pivot and use it to form a new tableau;	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Discerning and Joyful** By exploring the properties of linear programming models, students develop a discerning mindset and experience joy in finding efficient ways to allocate resources, plan production, or solve optimization problems. • Option for the Poor and Vulnerable • Call to Family, Community, and Participation	Linear programming (part 1) Matrices

		• be able to identify if a tableau satisfies the optimality condition. • know how to use slack and surplus variables; • understand and be able to use artificial variables; • be able to use the two-stage simplex algorithm; • be able to use the big-M method; • be able to relate the solution to the original problem. •			• Access to software or online tools that facilitate solving linear programming problems and analyzing the results. • Economics • Business Studies • Computer Science • Operations Research Analyst • Financial Analyst • Supply Chain Manager • Management Consultant • Data Scientist	
Critical path analysis (part 2)	• Resource histograms • Scheduling	• be able to draw and interpret resource histograms; • be able to level resource histograms. • be able to construct a scheduling diagram; • be able to interpret and modify schedules to meet requirements.	End of term assessment	Textbook/Practice Book/Other online resources These include: 1. Videos 2. Practice questions 3. Past exam questions 4. Opportunities for flipped learning	• **Listening and Attentive** By being attentive to the dependencies and constraints within a project, students can develop the skill of active listening and paying close attention to the critical elements that impact the overall timeline.	Critical path analysis (part 1)

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		•			• Call to Family, Community, and Participation • Option for the Poor and Vulnerable • Exposure to project management courses or workshops that cover critical path analysis in the context of planning and scheduling. • Business Studies • Economics • Design & • Technology • Project manager • Construction • manager • Event planner • Operations • manager • Supply chain • Manager • Civil Engineer	